Hysteresis Implies Scale Effects*

Conor Walsh†
Yale University

This Version: April 2020

Abstract

I show that if there are diminishing returns to research, long-run economic outcomes are unaffected by short-run fluctuations. Theories that feature hysteresis imply counterfactual scale effects. As such, the study of business cycles and long-run economic growth can be divorced.

Keywords: Growth, Fluctuations

JEL Codes: O40, E32

*I am indebted to Michael Peters for helpful discussions. The Washington Center for Equitable Growth provided valuable research support.

†Department of Economics; Email: conor.walsh@yale.edu

1. Introduction

Sustained economic growth is widely held to arise from the accumulation of knowledge. Part of this knowledge is developed by individuals in search of profits. Since profits are impacted by the business cycle, a natural question is whether downturns can have permanent effects on the level of output (otherwise known as *hysteresis*) via temporary reductions in innovative effort.

Take, for example, the procyclicality of measured R&D. Incumbent firms tend to concentrate their innovative efforts in booms, and cut back sharply in downturns.[1] Though the same is true for capital investment, innovative activity has effects not found in classical business cycle theory.

Knowledge is a unique production input, in that it is able to be used by others without loss to the inventor (Romer, 1990). That includes future researchers; knowledge won today can spill over through time, benefitting future innovation. Once one grants this premise, it is easy to imagine that a lack of innovation today could lower income in all future periods, even if the growth rate returns to normal after a downturn.

Many papers examining this possibility appear in the literature. Barlevy (2004) presents a growth model in which stabilizing temporary fluctuations can have permanent effects on the level of income, and uses it to revisit the costs of business cycles. Comin and Gertler (2006) show how high-frequency volatility can propagate onto medium and long-run economic dynamics through its effect on innovation and technology adoption. Benigno and Fornaro (2018) study the possibility of Keynesian "stagnation traps", where a period of low demand lowers the incentive for innovation, and weak growth depresses demand, permanently scarring output. Jordà et al. (2020) revisit the long-run neutrality of money, and overturn this result by including innovation and hysteresis in an otherwise standard New Keynesian model.[2]

I show in this paper that when there are diminishing returns to research, long-run levels of income are unaffected by short-run fluctuations. By diminishing returns, I mean that continued expansion of research effort in the aggregate is needed to keep delivering the same proportional growth in aggregate productivity. Evidence for diminishing returns is strong. Bloom et al. (Forthcoming) show that across domains as varied as transistors, agricultural yields and the invention of new drugs, increasing numbers of researchers have been needed to sustain proportional growth. At the macro level, a large rise in resources devoted to innovation in the 20th Century delivered approximately constant aggregate TFP growth (Jones, 1995b).

[1] See, for example, Ouyang (2011) and Argente et al. (2018). Several reasons for this behaviour have been proposed. Barlevy (2007) considers the role of business-stealing in innovation, which causes firms to concentrate their investments in booms when short-run profits are highest. Counter-cyclical credit constraints also seem to play a role (Aghion et al., 2012).

[2] Related papers exploring this idea include King and Rebelo (1988), Stadler (1990), Fatas (2000), Canton (2002), Aghion et al. (2010), Kung and Schmid (2015), Moran and Queralto (2018), Garga and Singh (2018), Queralto (2019), Bianchi et al. (2019) and Anzoategui et al. (2019). I discuss some of these in more detail in Section 3.

The reason for the disconnect between long-run income and short-run fluctuations is simple. In a world of linear returns to research, the productivity of an innovator in achieving a proportional increase to the knowledge stock is independent of the level of knowledge. If a downturn causes a dip in her research effort, that research time is lost forever, and output is permanently lower than it would have been. This same linearity is also what produces scale effects in growth models: more researchers means more ideas discovered, at all points in time.

If instead there are diminishing returns to research, long-run progress requires a growing numbers of researchers. If the potential population of researchers continues growing during a downturn, lost research time can be made up afterwards. In the long run, income is proportional only to the level of population, leaving no room for past fluctuations to matter.

I then show that papers who escape this conclusion rely on linearity in returns to research, in one way or another. In effect, they assume that a fixed amount of scare resources (such as labour for innovation) can produce perpetual growth in output. As a result, they imply strong scale effects, wherein increasing the population of innovators will raise the long-run growth rate. The post-war evidence of industrialized nations speaks strongly against the existence of such scale effects.

Finally, a word about what this paper is not about. I do not address hysteresis arising from a depressed labour market. Discussed in DeLong et al. (2012) and elsewhere, this could arise from skills atrophying during prolonged periods of unemployment, from information frictions or through psychological mechanisms. While such effects could be very persistent, they are not truly permanent in that workers are finitely lived, and are eventually replaced by younger cohorts with no memory of earlier downturns.

2. SHORT-RUN FLUCTUATIONS AND LONG-RUN GROWTH

The basic environment is simple. There is a representative consumer with intertemporal preferences over per-capita consumption c_t of a homogenous final good given by

$$U = \mathbb{E} \sum_{t=0}^{\infty} \beta^t L_t \frac{c_t^{1-\theta}}{1-\theta}.$$

The agent has an endowment of labour L_t that is growing by a proportional factor $(1+\eta)$ in each period, where $\eta \geq 0$.

The economy admits an aggregate production function for the final good of

$$Q_t = B_t A_t L_t^P.$$

Here A_t is the stock of knowledge, L_t^P is labour used for production, and B_t is a stochastic utilization factor that follows a Markov process. This production function is more general than it appears; a large class of models with different microfoundations can be shown to admit such an aggregate structure, some of which I explore in Section 3. The omission of physical capital from the function may strike some as odd, but its inclusion in this section changes nothing fundamental, while complicating the exposition.

Increasing A_t requires using labour for research, so that

$$A_{t+1} - A_t = \chi A_t^{\gamma} L_t^R, \tag{1}$$

where $\gamma \leq 1$, χ is a scale parameter and L_t^R is the amount of labour employed for research.

To begin with, I show how hysteresis may arise. Suppose that $\gamma = 1$, so that a proportional increase in knowledge today leads to a proportional increase in the productivity of research labour in the future. Under this restriction, we must have population growth $\eta = 0$, or the agent will be able to achieve unbounded utility. This restriction, while perhaps seeming a technicality, is at the heart of the issue. With linear returns to research, the growth rate of knowledge in (1) will depend only on the total amount of researchers employed at any time.

The problem of the agent is, recursively,

$$V(A, B) = \max_{R \in [0,1]} \frac{(BA(1-R))^{1-\theta}}{1-\theta} + \beta \mathbb{E}_{B'|B}[V((1+\chi RL)A, B')],$$

where R denotes the fraction of aggregate labour engaged in research. It is easy to verify that this problem admits a solution of the form $V = \frac{b(B)A^{1-\theta}}{1-\theta}$, where $b(B)$ is a function that solves

$$b(B)/(1-\theta) = \max_{R \in [0,1]} \left[(B(1-R))^{1-\theta} + (1+\chi RL)^{1-\theta} \beta \mathbb{E}_{B'|B}[b(B)] \right] / (1-\theta).$$

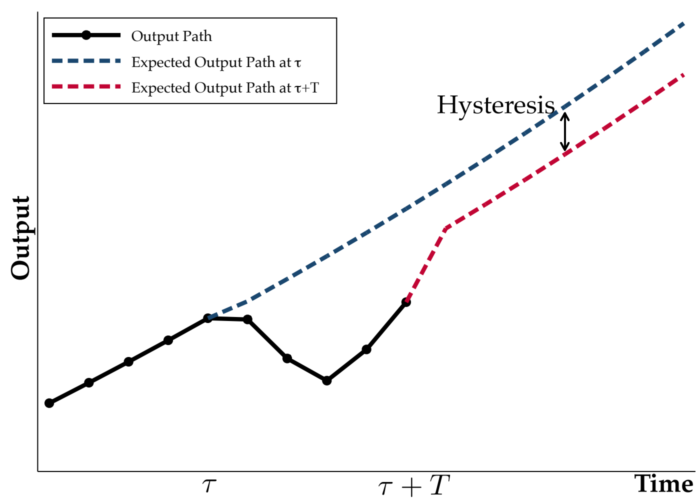
Moreover, optimal innovative effort R_t will solve

$$\frac{(1+\chi R_t L)^{\theta}}{(1-R_t)^{\theta}} = B_t^{\theta-1} \mathbb{E}_{B_{t+1}|B_t}[b(B_{t+1})] \beta \chi L. \tag{2}$$

The effect of fluctuations on innovation is easiest to see if we assume that B_t is i.i.d. In that case, R_t depends only on the current realization of B_t. With $\theta > 1$, innovative effort is lower when utilization B_t is lower. This reflects the fact that, even though the opportunity cost of doing research is lower with low B_t , sufficient curvature in utility means the agent partially offsets declines in current consumption through reduced innovative effort. The opposite occurs when $\theta < 1$.

This also means that research effort R_t tracks the process for B_t, and the growth rate of the economy will show no tendency to compensate for past slowdowns. An illustrative example is presented in

FIGURE 1: Hysteresis with Linear Returns To Knowledge



Notes: The Figure shows a stylized example of hysteresis occurring under linear returns to research, and i.i.d shocks to B_t. A sequence of bad shocks to utilization occurs when the black and blue lines diverge at τ. The red line shows the expected future output path at $\tau + T$, which is persistently below that expected at τ.

Figure 1. At time τ, the economy experiences a sequence of bad shocks to B_t, which directly lowers output, as well as slowing research effort. By time $\tau + T$, the expected growth path is permanently lower than that which would have been predicted at time τ, while expected growth rates and utilization are identical from then on. The expected knowledge stock at time t is

$$\mathbb{E}_t[A_{t+T}] = A_t \mathbb{E}_t \prod_{s=1}^{T} (1 + \alpha R_s L),$$

and with B_t i.i.d this reduces to

$$\mathbb{E}_t[A_{t+T}] = A_t (1 + \alpha \bar{R} L)^T, \tag{3}$$

where $\bar{R}$ is obtained from integrating the solution to (2) for R_t over the distribution of B_t. The expected knowledge stock at a future date is proportional to the level today. If A_t grows more slowly due to a shock to current B_t, the far future is permanently affected.[3] In other words, output is not trend stationary.

Under diminishing returns to research, this cannot happen. The economy completely recovers after bad (or good) shocks, and returns to a unique, long-run trend path. Suppose that $\gamma < 1$, and that $\eta > 0$. Then the percentage change in the stock of knowledge is

$$\frac{A_{t+1} - A_t}{A_t} = \chi R_t \frac{L_t}{A_t^{1-\gamma}}.$$

[3]Of course, the converse is also true, and booms will permanently raise living standards in all future time periods.

Writing it in this way, we can see that since R_t is bounded above by 1, long-run growth depends on a continued expansion in population; the diminishing returns to the effort of each researcher must be offset by continued growth in the number of researchers. This also suggests that the relevant state variable of the economy is not the amount of knowledge per se, but the ratio of knowledge to total available labour. I define this ratio to be $G_t \equiv A_t/L_t^{\frac{1}{1-\gamma}}$, which turns out to be the appropriate way to detrend aggregate productivity improvement. When this ratio is low, increasing the fraction of the population engaged in research yields high returns in knowledge gained, and vice versa.

Using this formulation allows us to rewrite the problem as

$$V(G,B) = \max_{R\in[0,1]} \frac{(BG(1-R))^{1-\theta}}{1-\theta} + \tilde{\beta}\mathbb{E}V(G',B'), \tag{4}$$

subject to

$$G' = (1+\eta)^{\frac{1}{\gamma-1}}[G + \chi R_t G^{\gamma}], \tag{5}$$

where $\tilde{\beta} \equiv \beta(1+\eta)^{\frac{2-\gamma-\theta}{1-\gamma}}$, and we impose $\tilde{\beta} < 1$. The state variable G plays a dual role in this formulation. It first captures the upwards drift of the population; without any research effort, G_t will decline at a constant rate. Second, G_t will be high when A_t is high, yielding increased possibilities for per-capita consumption relative to trend.

The Euler equation in this problem reads

$$\tilde{c}_t^{1-\theta}[1 + \chi R_t G^{\gamma-1}] = \tilde{\beta}\mathbb{E}\tilde{c}_{t+1}^{1-\theta}(G_t^{\gamma-1}(1-R_t)\chi), \tag{6}$$

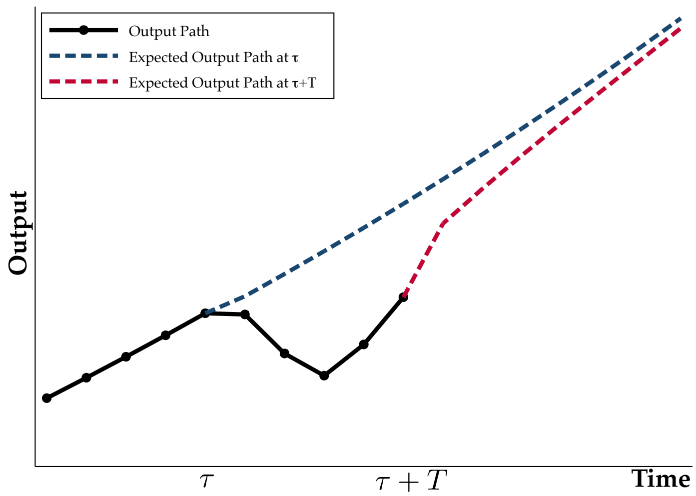
with $\tilde{c}_t \equiv B_t L_t^{-\frac{1}{1-\gamma}} A_t(1-R_t)$ being detrended per-capita consumption. Inspecting this equation, we can see that regardless of the process for B_t, the economy has a unique deterministic steady-state value for G_t. First note that if G_t is constant at $\bar{G}$, then A_t is growing at rate $(1+g_A) = (1+\eta)^{\frac{1}{1-\gamma}}$, and $\chi R_t \bar{G}^{\gamma-1} = g_A$. Using this in (6), we find that

$$\bar{G} = ((1+2g_A)^{-1}\chi\tilde{\beta})^{\frac{1}{1-\gamma}}.$$

This unique steady state is the point around which the economy fluctuates. Using standard techniques, it can be shown that the linearised dynamics of (5) and (6) around this steady state have a unique, non-explosive solution, and that $\mathbb{E}_t[G_{t+T}] \to \bar{G}$ as T grows large.[4] This means that in the long run, productivity A_t depends only on the level of population L_t, and there is no possibility of

[4]For the case of log presences with $\theta \to 1$, it can be shown that the economy is globally stable around $\bar{G}$. However, the separability implied by log preferences means that innovation is unaffected by fluctuations in B_t, and is thus not a terribly interesting case.

FIGURE 2: Convergence with Diminishing Returns To Knowledge



Notes: The Figure shows a stylized example of convergence occurring under diminishing returns to research, and i.i.d shocks to B_t. A sequence of bad shocks to utilization occurs when the black and blue lines diverge at time τ. The red line shows the expected future output path at $\tau + T$, which converges back to the expected output path at time τ.

hysteresis, in stark contrast to equation (3) above. Figure 2 displays an example.

Indeed, hysteresis can not arise even if temporary fluctuations destroy knowledge itself. We might imagine that a severe enough recession could destroy firms' organizational capital and accumulated inventions through waves of bankruptcies. In this case, A_t itself might fall. However, as long as the population of potential researchers is unaffected, eventually the economy will return to its long-run trend.

Why is diminishing returns so different? In effect, when the amount of resources available for innovation is high relative to the current state of knowledge, investing a greater fraction of these resources for the future yields a high return. After a downturn, this incentive drags the economy back to a trend path determined by the expansion of scarce resource availability.

3. Hysteresis and Scale Effects

Clearly, most papers studying growth and fluctuations are not this simple. Nonetheless, prominent formulations rely on linear returns to research effort to deliver long-run effects of temporary fluctuations. To show this, I introduce some minimal departures from the framework in Section 2 that nonetheless capture the key ingredients of papers in the literature.[5]

[5] Throughout this Section, I focus on the planner's problem in these economies. None of the conclusions change when examining the decentralized market equilibria, despite the presence of inefficiencies due to intertemporal spillovers.

3.1 AK and Lucas Models

Barlevy (2004) employs a classic "AK" model to study fluctuations and growth. The AK model is perhaps the earliest model of endogenous growth. In this world, production is linear in capital, and the classic convergent Solow dynamics do not apply; linear returns in the accumulating factor allows the economy to grow per-capita income forever.

While my discussion above features no capital, its setup is equivalent to an AK model if we simply relabel the knowledge stock A_t as capital. Then, following Barlevy, modify the knowledge accumulation equation according to

$$A_{t+1} - A_t = (\phi(\frac{I_t}{A_t}) - \delta)A_t, \tag{7}$$

where I_t is the amount of the final good used for investment, and ϕ is a increasing, concave function that introduces adjustment frictions. δ is a constant depreciation rate, which allows some knowledge to be forgotten every period. Letting $\phi()$ be the identity function would return us to the world of Section 2 with $\gamma = 1$, with the small change that now the utilization factor B_t also appears in the accumulation equation.

The linearity in returns to research here is readily apparent. Regardless of the scale of the economy, or the level of A_t, investing a constant fraction of output will deliver constant aggregate growth in income (on average). It can be verified that regardless of the function assumed for ϕ, hysteresis may arise. More importantly, just as in Section 2, the level of population L_t always matters for the equilibrium growth rate.

A related early endogenous growth model is Lucas' model of human capital accumulation. As is well known, the Lucas model is isomorphic to an AK model in many respects, and features linear returns to human capital accumulation at the aggregate level. King and Rebelo (1988) employ a two-sector Lucas model in an early model of endogenous growth and fluctuations, and note that "generally, there are permanent effects of temporary shocks".

3.2 Romer Models

A number of recent papers explicitly model knowledge accumulation through expansion of the *number* of ideas, as in the original Romer (1990). Specifically, consider a stripped-down, one sector version of Comin and Gertler (2006), where

$$Q_t = (\int_0^{A_t} y_t(i)^{\frac{\epsilon-1}{\epsilon}} di\Big)^{\frac{\epsilon}{\epsilon-1}},$$

with $\epsilon > 1$, such that final output is produced through aggregating intermediate varieties, and the output of each intermediate good is $y_t(i) = B_t k_t(i)^{\alpha} l_t(i)^{1-\alpha}$. $k_t(i)$ and $l_t(i)$ are capital and labour employed to produce intermediate i. By symmetry across goods, it can be shown that the economy admits an aggregate production function, such that

$$Q_t = A_t^{\frac{1}{\epsilon-1}} B_t K_t^{\alpha} L_t^{1-\alpha}.$$

Notice that the number of varieties A_t plays the role of the knowledge stock in this formulation, and continual growth in these varieties is the key to long-run growth in output. Ignoring their distinction between inventing a new variety and adopting it for production[6], assume that varieties accumulate according to

$$A_{t+1} = A_t(1-\delta) + \varphi_t S_t^{\rho}, \tag{8}$$

in which S_t is the amount of the final good spent on innovation, $\rho < 1$ and φ_t denotes research productivity, defined as

$$\varphi_t = \chi \frac{A_t}{K_t^{\rho}}, \tag{9}$$

where χ is a parameter. It is readily apparent that the knowledge accumulation equation (8) is linear in research spillovers (just as in (7) and (1) when $\gamma = 1$); the productivity of research investment S_t increases linearly in the current stock of knowledge. The role played by the scaling factor is K_t^{ρ} is less immediately clear. To see its effect, it is helpful to consider a world in which capital can be used in the same period it is produced, using only the final good. Maximising net output with respect to capital yields the first order condition $K_t = (\alpha A_t^{\frac{1}{\epsilon-1}} B_t)^{\frac{1}{1-\alpha}} L_t$, and net output becomes

$$Q_t = \bar{\alpha}(A_t^{\frac{1}{\epsilon-1}} B_t)^{\frac{1}{1-\alpha}} L_t,$$

with $\bar{\alpha} = \alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}} > 0$. Now growth in the knowledge stock takes the form

$$\frac{A_{t+1} - A_t}{A_t} = \bar{\chi}(R_{t,Q})^{\rho} - \delta,$$

where $R_{t,Q}$ is the fraction of output invested in research, and $\bar{\chi} = \chi(\alpha^{\frac{\alpha-1}{1-\alpha}} - 1)^{\rho}$. Written this way, we are very close to the world of Section 2. The problem of our agent is

$$V(A,B) = \max_{R_Q \in [0,1]} \frac{(B^{\frac{1}{1-\alpha}} A^{\epsilon}(1-R_Q))^{1-\theta}}{1-\theta} + \beta \mathbb{E}_{B'|B}[V((1-\delta+\bar{\chi}R_Q^{\rho})A, B'),$$

with $\epsilon \equiv (1-\alpha)^{-1}(1-\epsilon)^{-1}$. It can be shown that in this case, the value function again has a closed

[6]This merely introduces a time delay between innovation and its effect on productivity, and is a way of generating procyclical propagation of shocks to B_t.

form solution, and output growth displays hysteresis; the process for output is non-stationary, and past slowdowns will lower the level of income for all time. In contrast to Section 2, however, the level of population does not appear in the research choice of the agent, thanks to the introduction of the scaling factor K_t in (9). Nonetheless, the economy displays a secondary type of scale effect. Changes in the average *fraction* of output invested in innovation (thanks, say, to R&D subsidies in a decentralised equilibrium) will permanently change the average growth rate. This type of scale effect is also rejected by the evidence of the 20th Century.

Papers which employ a Romer style formulation similar to Comin and Gertler (2006) include Moran and Queralto (2018), Queralto (2019) and Anzoategui et al. (2019).

3.3 Schumpeterian Models

Schumpeterian models consider innovation occurring on a range of established goods (or industries), and improvements in knowledge are best thought of as improvements in the *quality* of existing ideas. As in Benigno and Fornaro (2018), suppose that output is produced according to

$$Q_t = (B_t L_t)^{1-\alpha} \int_0^1 A_t(i)^{1-\alpha} x_t(i)^{\alpha} di,$$

where $A_t(i)$ is the quality of good i, and $x_t(i)$ is the amount of intermediate i used. Intermediates can be produced using final output with a unit cost. This economy admits a familiar aggregate production function, given by

$$Q_t = \bar{\alpha} B_t A_t L_t,$$

where A_t is now the average quality of each good across product lines. In these models, good quality is modeled as lying on a ladder with a proportional step size $\gamma > 1$, such that

$$A_t(i) = \gamma^{n_t(i)},$$

and $n_t(i)$ is the number of previous successful innovations. Improving a particular product requires investment in research, and is stochastic, with the chance a product improves after investing $I_t(i)$ units of the final good given by

$$\mu_t(i) = min\{\frac{I_t(i)}{A_t(i)}, 1\}.$$

Importantly, innovation on a product becomes less likely the more advanced a product is, and requires spending larger amounts of the final good. As is standard in the literature, it can be shown that there is a symmetric solution where $\mu_t(i)$ is constant across goods, so that average quality evolves according to

$$A_{t+1} = (1 - \mu_t)A_t + \gamma \mu_t A_t,$$

or equivalently

$$A_{t+1} - A_t = \gamma A_t B_t L_t^R,$$

where again L_t^R is the number of researchers engaged in research activity. The parallel with Section 2 should now be obvious: the knowledge accumulation equation is linear in research spillovers, and output growth will again be non-stationary, with hysteresis a feature.

The linearity in research spillovers in the aggregate economy of Schumpeterian models arises from their quality-ladder structure. Each step increases productivity by a fixed proportional factor, and simultaneously increases the knowledge base for future researchers on that product by the same factor. Besides Benigno and Fornaro (2018), papers in the literature with a Schumpeterian structure include Fatas (2000), Aghion et al. (2010) and Bianchi et al. (2019).

4. Conclusion

Early business cycle theorists modeled the economy as fluctuating around an exogenous trend driven by technological progress, reasoning that fluctuations were unlikely to permanently affect that progress. After three decades of work on the empirics and theory of endogenous growth, that assumption looks justified.

Of course, the long-run invariance result here says nothing about the length (or pain) of downturns, which could be exacerbated in the short-run by innovation slumps. As is known from Jones (1995a) and Atkeson and Burstein (2019), when γ is close to 1, transitional dynamics can be slow. Getting a handle on these dynamics in a world of diminishing returns is a key task for future research.

References

Aghion, Philippe, George-Marios Angeletos, Abhijit Banerjee, and Kalina Manova, "Volatility and Growth: Credit Constraints and the Composition of Investment," *Journal of Monetary Economics*, 2010, *57* (3), 246–265.

__ , Philippe Askenazy, Nicolas Berman, Gilbert Cette, and Laurent Eymard, "Credit Constraints and the Cyclicality of R&D Investment: Evidence from France," *Journal of the European Economic Association*, 2012, *10* (5), 1001–1024.

Anzoategui, Diego, Diego Comin, Mark Gertler, and Joseba Martinez, "Endogenous Technology Adoption and R&D as Sources of Business Cycle Persistence," *American Economic Journal: Macroeconomics*, 2019, *11* (3), 67–110.

Argente, David, Munseob Lee, and Sara Moreira, "Innovation and Product Reallocation in the Great Recession," *Journal of Monetary Economics*, 2018, *93*, 1–20.

Atkeson, Andrew and Ariel Burstein, "Aggregate Implications of Innovation Policy," *Journal of Political Economy*, 2019, *127* (6), 2625–2683.

Barlevy, Gadi, "The Cost of Business Cycles Under Endogenous Growth," *American Economic Review*, 2004, *94* (4), 964–990.

__ , "On the Cyclicality of Research and Development," *American Economic Review*, 2007, *97* (4), 1131–1164.

Benigno, Gianluca and Luca Fornaro, "Stagnation Traps," *The Review of Economic Studies*, 2018, *85* (3), 1425–1470.

Bianchi, Francesco, Howard Kung, and Gonzalo Morales, "Growth, Slowdowns, and Recoveries," *Journal of Monetary Economics*, 2019, *101*, 47–63.

Bloom, Nicholas, Charles I Jones, John Van Reenen, and Michael Webb, "Are Ideas Getting Harder to Find?," *American Economic Review*, Forthcoming.

Canton, Erik, "Business Cycles in a Two-sector Model of Endogenous Growth," *Economic Theory*, 2002, *19* (3), 477–492.

Comin, Diego and Mark Gertler, "Medium-Term Business Cycles," *American Economic Review*, 2006, *96* (3), 523–551.

DeLong, J Bradford, Lawrence H Summers, Martin Feldstein, and Valerie A Ramey, "Fiscal Policy in a Depressed Economy," *Brookings Papers on Economic Activity*, 2012, pp. 233–297.

Fatas, Antonio, "Do Business Cycles Cast Long Shadows? Short-run Persistence and Economic Growth," *Journal of Economic Growth*, 2000, *5* (2), 147–162.

Garga, Vaishali and Sanjay R Singh, "Output Hysteresis and Optimal Monetary Policy," *Working Paper*, 2018.

Jones, Charles I, "R&D-based Models of Economic Growth," *Journal of Political Economy*, 1995, *103* (4), 759–784.

_ , "Time Series Tests of Endogenous Growth Models," *The Quarterly Journal of Economics*, 1995, *110* (2), 495–525.

Jordà, Òscar, Sanjay R Singh, and Alan M Taylor, "The Long-run Effects of Monetary Policy," 2020.

King, Robert and Sergio Rebelo, "Business Cycles with Endogenous Growth," 1988.

Kung, Howard and Lukas Schmid, "Innovation, Growth, and Asset prices," *The Journal of Finance*, 2015, *70* (3), 1001–1037.

Moran, Patrick and Albert Queralto, "Innovation, Productivity, and Monetary Policy," *Journal of Monetary Economics*, 2018, *93*, 24–41.

Ouyang, Min, "On the Cyclicality of R&D," *Review of Economics and Statistics*, 2011, *93* (2), 542–553.

Queralto, Albert, "A Model of Slow Recoveries from Financial Crises," *Journal of Monetary Economics*, 2019.

Romer, Paul M, "Endogenous Technological Change," *Journal of Political Economy*, 1990, *98* (5 pt 2).

Stadler, George W, "Business Cycle Models with Endogenous Technology," *The American Economic Review*, 1990, pp. 763–778.